Quarter-Wave Coupled Junction Circulators Using Weakly Magnetized Disk Resonators

JOSEPH HELSZAJN, MEMBER, IEEE

Abstract —The simple classic theory of very weakly magnetized junction circulators using disk resonators indicates that the loaded Q-factor of the junction is completely determined by the magnetic variables of the garnet or ferrite resonator. An exact theory using numerical methods suggests that this approximation is nearly met for all values of the coupling angle ψ sustended by the striplines at the resonator terminals, provided the ratio of the off-diagonal and diagonal elements K and μ of the tensor permeability has an upper bound of about 0.30. The corresponding minimum realizable value of the loaded Q-factor is approximately 2. This value of loaded Q-factor is suitable for the realization of quarter-wave coupled junction circulators with bandwidths of 25 percent at the 20-dB frequencies or 18 percent at the 26-dB frequencies. The improved closed form solution is based on a seven-mode description of this class of circulator is obtained in this paper by adding a correction polynomial to the classic very weakly magnetized formulation.

I. Introduction

THE THEORY OF planar junction circulators using very weakly magnetized disk resonators indicates that the loaded Q-factor Q_L of the junction is completely determined by the ratio of the off-diagonal and diagonal elements K and μ of the tensor permeability [1], [2]. However, the range of applicability of this relationship has on occasion been overestimated [3]–[7]. A complete description of the junction circulator using disk resonators indicates that this classic approximation is nearly met with K/μ between 0 and 0.30, but that it displays significant deterioration at K/N equal to 0.35. The discrepancy between the exact and the very weakly magnetized theories is accounted for in this paper by introducing a second-order polynomial correction factor to both the loaded Q-factor and susceptance slope parameter of the junction. The gyrator conductance is assumed sufficiently well described by the very weakly magnetized case. This approximate closed-form solution is in excellent agreement with the exact one except for a slight shift in the assumed value in the resonator cutoff number $kR = 1.84$. The result in this paper has been obtained by retaining the first seven resonator modes in the description of the complex gyrator circuit of the junction [10]. The lower bound on Q_L is obtained from this work as about 2. Such a value of Q_L is consistent with the realization of junction circulators with bandwidths of 25 percent at the 20-dB points (or bandwidths of 18 percent at the 26-dB points). In addition to a knowledge of Q_L, it is also necessary to ensure that the equivalent circuit of the junction is well behaved over the frequency range of the specification [8]. This condition is sufficiently well described for practical purposes for below resonance circulators with Q_L equal to or greater than 2.

This paper includes the synthesis of junction circulators for which the dielectric constant of the transformer region is chosen as an independent variable. A special example of this situation is the realization of junction circulators using a single garnet substrate for both the resonator and quarter-wave transformers.

Low magnetization materials may be used to advantage in the design of high-peak power devices or in the realization of millimeter devices.

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II. Loaded Q-Factor of Weakly Magnetized Junction Circulators Using Disk Resonators

The accurate design of quarter-wave coupled junction circulators requires a knowledge of the loaded Q-factor of the junction. This quantity has been obtained in this paper by forming the complex gyrator circuit of the junction in the vicinity of the Davies and Cohen first circulation condition (using the first seven modes) [10]. It is displayed in Table I for K/μ between 0 and 0.40 and parametric values of the stripline coupling angle ψ (defined in Figs. 1 and 2). This result indicates that Q_L is nearly independent of ψ for K/μ between 0 and 0.30. In this interval, Q_L may be described by a polynomial approximation in terms of K/μ

$$Q_L \approx 0.689\frac{\mu}{K} + \left(0.0463 - 2.6318\frac{K}{\mu} + 3.5513\frac{K^2}{\mu^2}\right), \qquad 0 \leqslant \frac{K}{\mu} \leqslant 0.30. \quad (1)$$

This polynomial is obtained by adding a correction polynomial of second order to the classic approximation for Q_L [1], [2]. This correction factor is derived by the method of least squares for the average value of Q_L over the variable ψ for the interval 0.10 to 0.70. The error in the classic term is about 2 percent at $K/\mu = 0.1$, increasing to 10 percent, 16 percent, and 23 percent at $K/\mu = 0.20$, 0.25, and 0.30. The correction factor is derived for K/μ between 0.05 and 0.30 using five points.

In addition to a description of the loaded Q-factor, it is also necessary to ensure that the equivalent circuit of the junction be well behaved over the frequency interval of the

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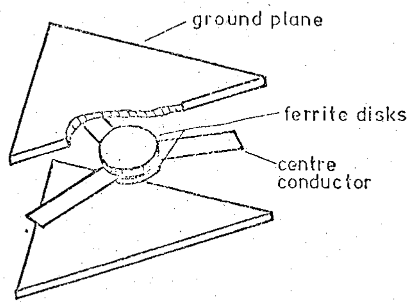


Fig. 1. Schematic diagram of stripline junction circulator.

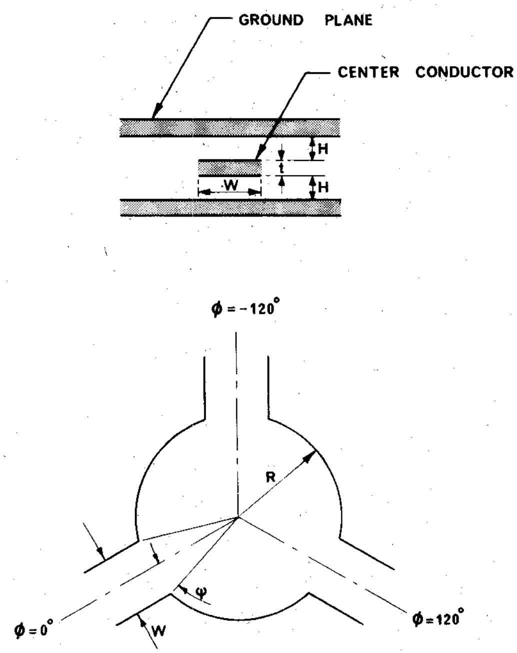


Fig. 2. Schematic diagram of stripline junction.

TABLE I
Q_L VERSUS ψ AND K/μ

K/μ psi	0.05	0.10	0.20	0.25	0.30	0.35	0.40
0.1	13.72	6.728	3.139	2.444	2.150	2.538	4.494
0.2	13.72	6.721	3.107	2.372	1.969	2.085	3.551
0.3	13.71	6.714	3.077	2.302	1.788	1.548	2.155
0.4	13.55	6.713	3.066	2.273	1.708	1.277	0.9912
0.5	13.71	6.689	3.077	2.291	1.733	1.305	0.9539
0.6	13.72	6.723	3.100	2.330	1.796	1.403	1.106
0.7	13.72	6.728	3.118	2.360	1.843	1.472	1.197

specification. Although this condition is not generally met, it is in fact satisfied in the design of junction circulators bounded by the values of loaded Q-factor in Table I for K/μ between 0 and 0.30 and biased below the main ferrimagnetic resonance.

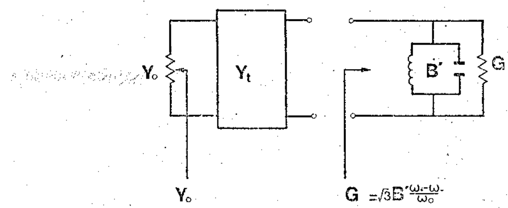


Fig. 3. Schematic diagram of quarter-wave coupled junction circulator.

III. Network and Microwave Variables of Junction Circulators Using Weakly Magnetized Disk Resonators

The synthesis of a quarter-wave coupled junction circulator in terms of a microwave specification is a straightforward task, providing its complex gyrator circuit may be represented by a frequency independent gyrator conductance in shunt with a distributed quarter-wave-long resonator. The relationship between the specifications of the overall device in terms of its VSWR r (or return loss) and bandwidth and the gyrator network in terms of its conductance G, susceptance slope parameter B' and loaded Q-factor Q_L is then described by the following standard equations [6], [9][1]:

$$G=\frac{Y_0(r-\sin^2\theta)}{r\cos^2\theta} \tag{2}$$

$$B'=\frac{\pi Y_0}{4}\cdot\frac{(r-\sin^2\theta)}{r\cos\theta}^{1/2}(r-1)\tan^2\theta \tag{3}$$

$$Q_L=\frac{B'}{G}=\left(\frac{\pi}{4}\right)\frac{(r-1)\sin\theta\cdot\tan\theta}{(r-\sin^2\theta)^{1/2}} \tag{4}$$

where

$$\cos\theta=\frac{1}{\sqrt{2}}\cos\theta_0 \tag{5}$$

$$\theta_0=\frac{\pi}{2}(1+\delta_0) \tag{6}$$

$$2\delta_0=\frac{\omega_2-\omega_1}{\omega_0}. \tag{7}$$

Here δ_0 is a normalized bandwidth parameter, ω_0 is the radian center frequency, $\omega_{1,2}$ are the bandedge frequencies, and Y_0 is 0.02 ℧.

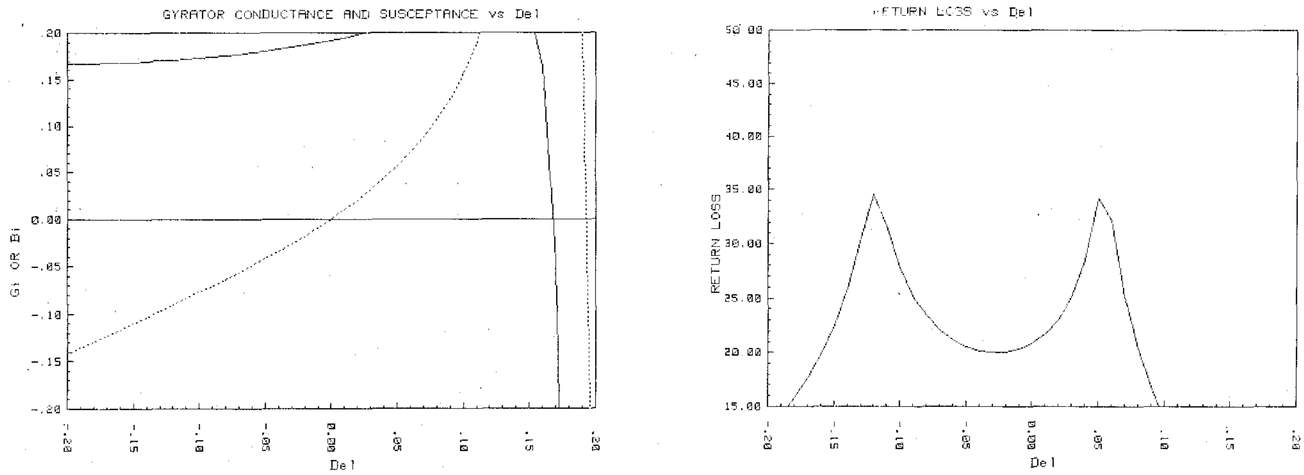
The admittance Y_t of the quarter-wave transformer is given by

$$Y_t^2=rG\cdot Y_0. \tag{8}$$

The single most important quantity in the description of the gyrator circuit is the realizable value of loaded Q-factor. Equation (4) indicates that for a constant value of Q-factor it is possible to accommodate a wide choice of VSWR and bandwidth variables. The 1-port equivalent circuit considered here is depicted in Fig. 3.

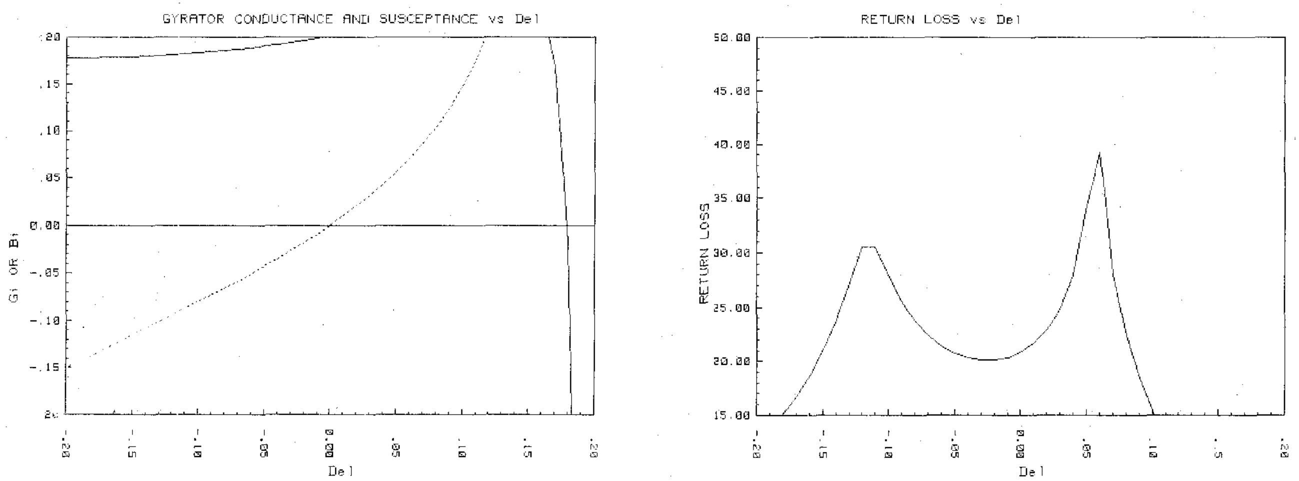
In the case of a junction circulator using a very weakly magnetized disk resonator, the real and imaginary parts G and B of the complex gyrator admittance, the susceptance

[1]In [9], θ_0 should be replaced by θ in (50) and (51).



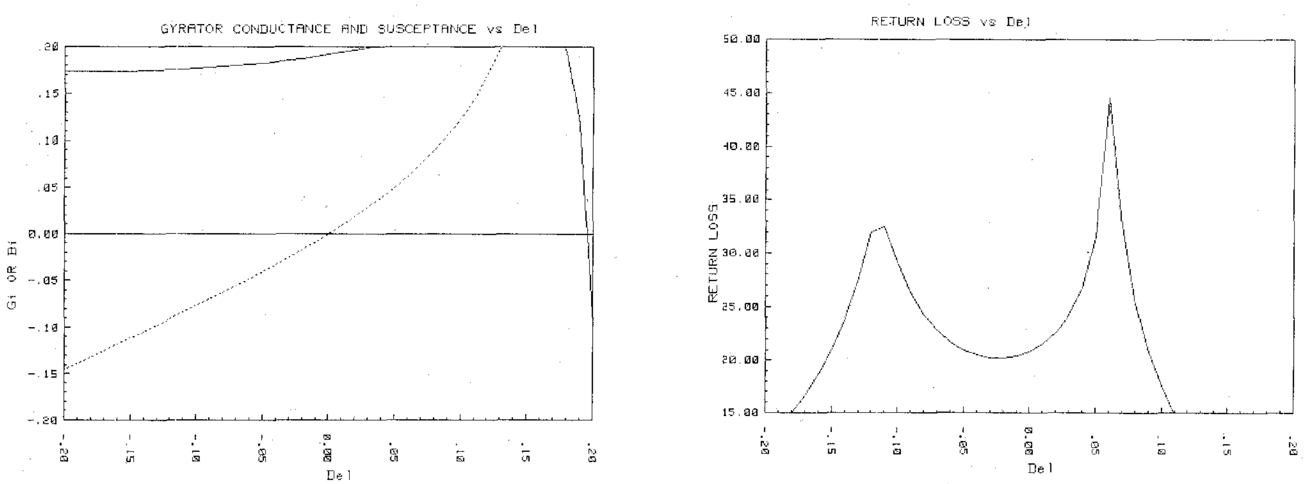
(a) (b)

Fig. 4 (a) Complex gyrator admittance of stripline junction for $\psi = 0.10$ and $K/\mu = 0.25$. (b) Return loss of quarter-wave coupled stripline junction circulator with $r = 1.20$, $\psi = 0.10$, and $K/\mu = 0.25$.



(a) (b)

Fig. 5(a) Complex gyrator admittance of stripline junction for $\psi = 0.20$ and $K/\mu = 0.25$. (b) Return loss of quarter-wave coupled stripline junction circulator with $r = 1.20$, $\psi = 0.20$, and $K/\mu = 0.25$.



(a) (b)

Fig. 6(a) Complex gyrator admittance of stripline junction for $\psi = 0.30$ and $K/\mu = 0.25$. (b) Return loss of quarter-wave coupled stripline junction for $r = 1.20$, $\psi = 0.30$, and $K/\mu = 0.25$.

slope parameter B', and the loaded Q-factor Q_L are related to the physical variables of the junction by Bosma's classic relationships [1]

$$G=\frac{\pi Y_f}{\sqrt{3}\sqrt{\mu_{\text{eff}}}(kR)\sin\psi}\frac{K}{\mu} \tag{9}$$

$$B=\frac{\pi Y_f}{3\sqrt{\mu_{\text{eff}}}\sin\psi}\left[\frac{J_1'(kR)}{J_1(kR)}\right] \tag{10}$$

$$B'=\frac{\pi Y_f}{3\sqrt{\mu_{\text{eff}}}\sin\psi}\left[\frac{(kR)^2-1}{2kR}\right] \tag{11}$$

$$Q_L\approx\left[\frac{(kR)^2-1}{2\sqrt{3}}\right]\frac{\mu}{K}. \tag{12}$$

The resonant frequency is determined by

$$kR=1.84. \tag{13}$$

The coefficient of μ/K in (12) reduces to the first term in (1).

The physical variables in (9)–(13) are defined by

$$\sin\psi=\frac{W}{2R} \tag{14}$$

$$k=\frac{2\pi}{\lambda_0}\sqrt{\epsilon_f\mu_{\text{eff}}} \tag{15}$$

$$Y_f=\sqrt{\epsilon_f}\,Y_r \tag{16}$$

$$Y_r=\left[30\pi\ln\frac{(W+t+2H)}{W+t}\right]^{-1}. \tag{17}$$

W, t, and H are defined in Fig. 2. For a saturated material with the internal direct field equal to zero

$$\mu_{\text{eff}}=1-p^2 \tag{18}$$

$$K=p \tag{19}$$

$$\mu=1 \tag{20}$$

$$p=\frac{\gamma M_0}{\mu_0\omega}. \tag{21}$$

The transformer admittance in (8) is described by

$$Y_t=\sqrt{\epsilon_t}\,Y_r \tag{22}$$

where γ is the gyromagnetic ratio (2.21×10^5 (rad/s)/(A/m)), M_0 is the saturation magnetization (T), μ_0 is the free-space permeability ($4\pi\times10^{-7}$ H/m), ω is the radian frequency (rad/s), λ_0 is the free-space wavelength (m), ϵ_f is the dielectric constant of the ferrite material, and ϵ_t is the dielectric constant of the transformer region.

The applicability of the preceding equations may be extended by replacing Q_L in (12) by the semiempirical form in (1) and by modifying the susceptance slope parameter in (11) accordingly. In the approximation employed here, it is found that the gyrator conductance is sufficiently well described by the form in (9). One suitable approximation for B' is therefore obtained by combining Q_L in (1) with G in (9)

$$B'\approx\frac{\pi Y_f}{\sqrt{3}\sqrt{\mu_{\text{eff}}}(kR)\sin\psi}\cdot\left\{\frac{(kR)^2-1}{2\sqrt{3}}+0.0463\frac{K}{\mu}-2.6318\frac{K^2}{\mu^2}+3.5513\frac{K^3}{\mu^3}\right\}. \tag{23}$$

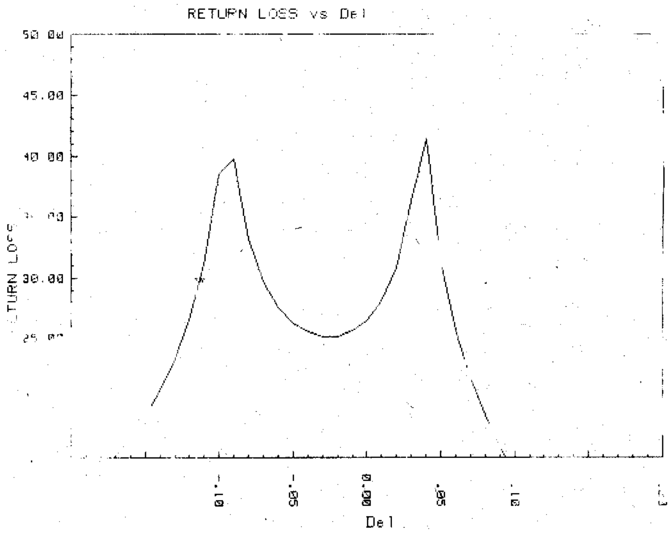


Fig. 7. Return loss of quarter-wave coupled stripline junction circulator with $r=1.10$, $\psi=0.20$, and $K/\mu=0.25$.

If ψ (or W) is taken as the independent variable, the network problem may be satisfied by using Y_r (or H) to set the absolute level of B' and G, and ϵ_t may be utilized to satisfy Y_t in (8). These two realizations lead to the Bosma or Fay and Comstock situations [1], [2].

Figs. 4, 5, and 6 depict the exact frequency responses of quarter-wave coupled junction circulators with K/μ taken as 0.25 and ψ equal to 0.10 rad, 0.20 rad, and 0.30 rad. This value of K/μ is compatible with the realization of junction circulators with $r=1.20$ and $2\delta_0=0.25$ ($Q_L=2.366$). These results are obtained by experimentally adjusting the absolute values of G and B' by varying Y_r. In keeping with the theory outlined in this paper, there is little to choose between these different designs in so far as their frequency responses are concerned. However, each one leads to a different value for Y_r (or H) and dielectric constant for the transformer region ϵ_t.

For the example employing $\psi=0.20$, the exact and calculated (in brackets) physical variables are described by

$$G=0.2019\ \mho \quad (0.2032)$$

$$B'=0.4789 \quad (0.4808)$$

$$Q_L=2.372 \quad (2.366).$$

For completeness, the real and imaginary parts of the complex gyrator circuit are also shown in Figs. 4, 5, and 6. Fig. 7 depicts a similar result but with $r=1.10$. Although the frequency behavior of the complex gyration circuits are not ideal, they appear to be adequate for the design of the class of quarter-wave coupled junction circulators discussed in this paper.

It is observed in this synthesis procedure that the dielectric constant of the transformer region is not necessarily an integer value. This difficulty may be overcome by choosing the dielectric constant as the independent variable or by altering the ripple level. The latter approach is dealt with in the next section.

IV. Synthesis of Junction Circulators Using Weakly Magnetized Disk Resonators and Transformers on Single Garnet Circuits

The fact that for weakly magnetized resonators the loaded Q-factor is independent of the physical variables of the junction allows either the constitutive parameters of the transformer region, the coupling angle ψ of the striplines, or the ground plane spacing to be used as the independent variable. By choosing the constitutive parameters of the transformer region as the independent variable, it is possible to accurately design junction circulators with both resonator and matching network on a single garnet or ferrite substrate.

The synthesis may be developed by first combining (8) and (22). The result is

$$Y_r = \sqrt{\frac{\mu_t r G Y_0}{\epsilon_t}}. \tag{24}$$

In obtaining this result, (22) has been modified to account for the permeability μ_t of the transformer region. ψ may be obtained from either G or the corrected form for B'. Taking the former relationship leads to

$$\sin\psi = \frac{\pi Y_r \sqrt{\epsilon_f}}{\sqrt{3}\sqrt{\mu_{\text{eff}}}(kR)G} \cdot \frac{K}{\mu}. \tag{25}$$

In the synthesis problem, the realization of the network parameters usually start with a specification in terms of the VSWR and bandwidth. Taking the value of Q_L in (1) corresponding to $K/\mu = 0.25$ gives

$$Q_L = 2.366$$
$$r = 1.20$$
$$2\delta_0 = 0.25.$$

G and B' are given from (1) and (3) by

$$G = 0.2032$$
$$B' = 0.4808.$$

The material parameters are in this example taken as

$$\frac{K}{\mu} = 0.25$$
$$\epsilon_f = 14.5$$
$$\mu_{\text{eff}} = 0.9375$$
$$\epsilon_t = \epsilon_f = 14.5$$
$$\mu_t = \mu_{\text{eff}} = 0.9375.$$

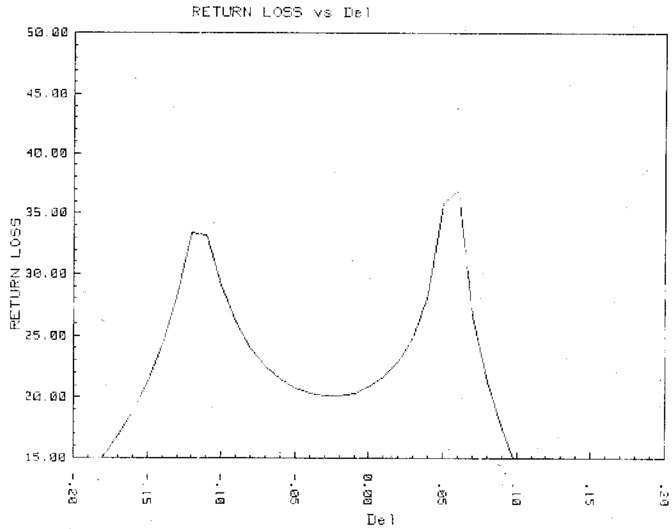


Fig. 8. Return loss of quarter-wave coupled stripline junction for $r = 1.20$, $\psi = 0.084$, and $K/\mu = 0.25$.

Y_r is now evaluated using (24) as

$$Y_r = 0.0177.$$

ψ is given from (25) by

$$\psi = 0.0849.$$

Finally, R is determined using $kR = 1.84$ (although some experimental adjustment of this quantity may be necessary), W is determined from a knowledge of ψ and R using (14), and H is calculated in terms of Y_r and W from (17).

Equatiof (25) may also be written in terms of H by combining (14), (17), and (24) with $t = 0$

$$2H = \frac{\pi Y_r \sqrt{\epsilon_f}}{\sqrt{3}\sqrt{\mu_{\text{eff}}} kG}\left[\text{antiln}\left(\frac{1}{30\pi Y_r}\right) - 1\right]\frac{K}{\mu}. \tag{26}$$

In this instance, W is determined in terms of Y_r and H, and R is obtained using $kR = 1.84$. Fig. 8 illustrates the exact frequency response for a device with the above physical variables except that kR is taken as $kR = 1.92$ (exact solution) instead of the value of 1.84 indicated by the simple theory.

V. Experimental Evaluation of Loaded Q-Factor Using Loosely Coupled Resonators

The loaded Q-factor of a junction circulator using a weakly magnetized resonator described by (4) or (12) is often expressed in terms of the split frequencies $\omega_{\pm}$ of the dominant $n = \pm 1$ modes of the magnetized resonator by

$$Q_L = \left[\sqrt{3}\left(\frac{\omega_+ - \omega_-}{\omega_0}\right)\right]^{-1}. \tag{27}$$

Whereas the preceding equation is only valid for p between zero and about 0.25 or 0.30, an experimental study of $\omega_{\pm}$ on loosely coupled magnetized disk resonators indicates that the splitting between the degenerate modes is well behaved up to at least p equal to 0.55.

Since the relationship between the magnetization and the

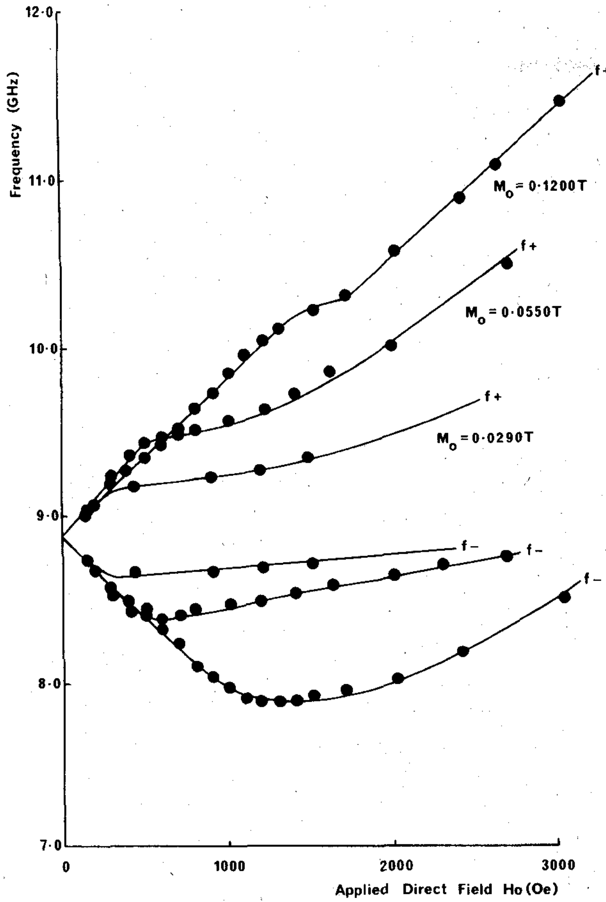


Fig. 9. Split frequencies of loosely coupled magnetized disk resonators ($M_0 = 0.0290$ T, 0.0550 T, 0.1200 T).

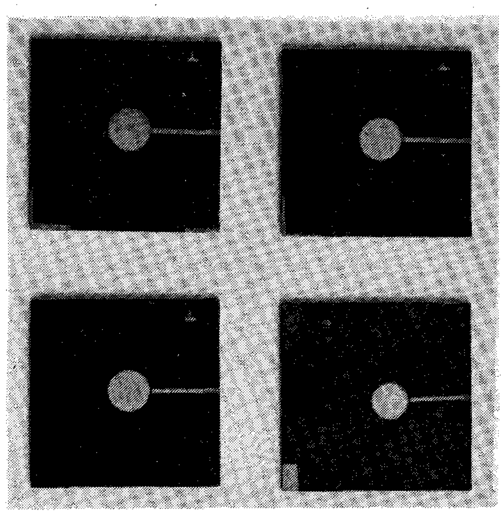


Fig. 10. Schematic diagram of loosely coupled disk resonators.

ratio of the off-diagonal and diagonal elements of the tensor permeability in a partially magnetized material is a difficult problem, (27) was verified experimentally at saturation using a number of materials with different values of saturation magnetization (0.0290 T, 0.0550 T, 0.0800 T, 0.1200 T, and 0.1750 T). Fig. 9 depicts the split frequencies for three values of magnetization. Similar measurements on the other two materials are omitted from this

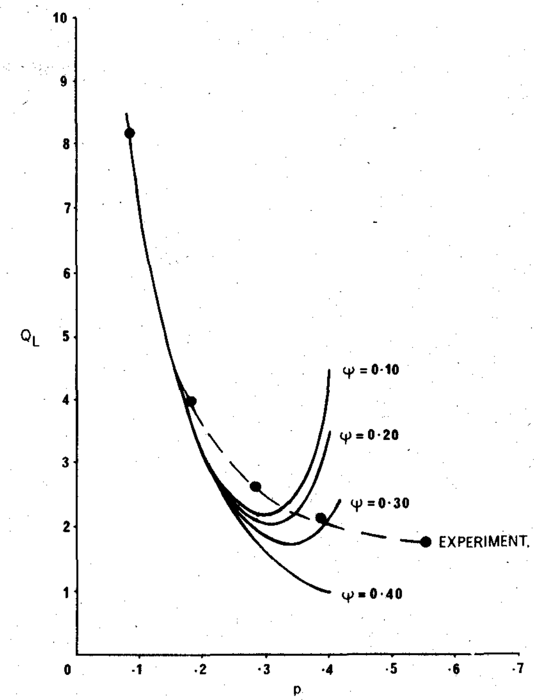


Fig. 11. Theoretical and experimental (based on approximate theory) loaded Q-factors of junction circulator using disk resonators.

illustration for clarity. Typical substrates are illustrated in Fig. 10. This result indicates that the initial splitting for each of these materials is a constant in agreement with theory. In order to account for saturation effects (in a magnetized substrate) in measuring the split frequencies of the magnetized resonator for use with (27), the deviation between the split frequencies at the origin was extended to the value of direct magnetic field at saturation ($H_0 \approx N_z M_0/\mu_0$) with $N_z = 1$. Fig. 11 compares the experimental loaded Q-factor obtained in this way with that determined using the more complete 7 modes theory summarized in Table I.

VI. Conclusions

The applicability of the classic closed-form solution on planar junction circulators using very weakly magnetized resonators has been evaluated in this paper using an exact numerical method. The classic description is shown to be approximately met provided the ratio of the off-diagonal and diagonal elements of the tensor permeability has an upper bound of 0.30. The discrepancy between the exact and approximate solutions is reconciled by introducing a polynomial correction factor in the description of the loaded Q-factor and susceptance slope parameter of the classic functions. No correction term is necessary in the description of the gyrator conductance.

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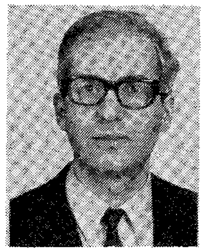

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Equivalent Circuit Representation for the E-Plane Circulator

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***Abstract*—The resonator structure employed in an E-plane circulator may be described by a series-resonant circuit as opposed to a parallel circuit in the case of comparable H-plane devices. This is shown to result in a change of the direction of circulation at the edges of the circulator passband. Employing the equivalent circuit representation methods to increase the bandwidth of the E-plane circulator are discussed.**

I. Introduction

During the last years, millimeter-wave E-plane (fin-line) circuits have been developed to a high degree of sophistication. Today, completely integrated RF front ends are available in fin-line technique. Since fin-line is basically an E-plane waveguide, where only E-plane junctions may be easily realized, circulators compatible with E-plane geometry are needed for more complex systems. This leads to new efforts in the investigation of E-plane circulators.

Up until now, E-plane circulators have been used mainly in high-power applications [1]–[3]. Other workers employ E-plane devices in an effort towards miniaturization and compactness [4], [5]. Neither in these papers nor in the only published field theoretical analysis of E-plane circulators [6], [7] has there been an analysis of the type of resonant structure employed in the junction circulators. Equally, there is no discussion of the differences in the behavior of the E-plane circulator as compared to the H-plane circulator.

In this paper the basic behavior of an E-plane Y-junction circulator with an unsymmetric ferrite insert employing the lowest order resonant mode is contrasted to the behavior of the commonly used H-plane devices, and an equivalent circuit representation for the E-plane circulator is presented which differs from that of the H-plane circulator.

II. The Resonant Structure (Lowest Order Mode)

The basic structure in Y-junction circulators is the resonator containing the anisotropic material (ferrite). In the following, the ferrite resonator is assumed to be a purely dielectric structure (zero magnetic bias field).

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